元気なうちに伝えたいこと、

「撓みに注目した最適化」

2025.09.29

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A-Forum
東工大名誉教授

Prof. Jerome. J. Connor, MIT said.

"First,
Design an elastic rational skeleton.

Second,
Dampers can make it a good structure."

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1. *Details*
2. *Deflection*
3. *Stress and Strength*

単 位 荷 重 法

上記が既知として、単位荷重法によって右端の変位を求める

「外荷重による変位」と「外荷重によるロッドの伸び」を一つの仮想変位のセットと考える

単位荷重　**1 x Δ = 4 x (60 x L / AE) = 4 x (60x100/100,000) = 0.24cm**

単位荷重による赤い棒の軸力

ロッドの伸び

外荷重によりロッドが伸び、
梁の先端に生じる変位が求まる

Unit Load Method for Calculating the Displacement of Structures

$$1 \times D = \int \frac{N_U N_L}{EA} dx + \int \frac{M_U M_L}{EI} dx + \int \frac{\alpha V_U V_L}{GA} dx$$

"The unit-load method can be used not only for beams, trusses, and other simple kinds of structures, but also for very complicated structures having many members. Furthermore, the unit-load method is suitable for finding all types of displacements, including the deflection of a point in the structure, the rotation of the axis of a member, the relative displacement between two points, and others. Theoretically, it may be used for either statically determinate or indeterminate structures, although for practical purposes the method is limited to determinate structures because its use requires that the stress resultants be known throughout the structure."

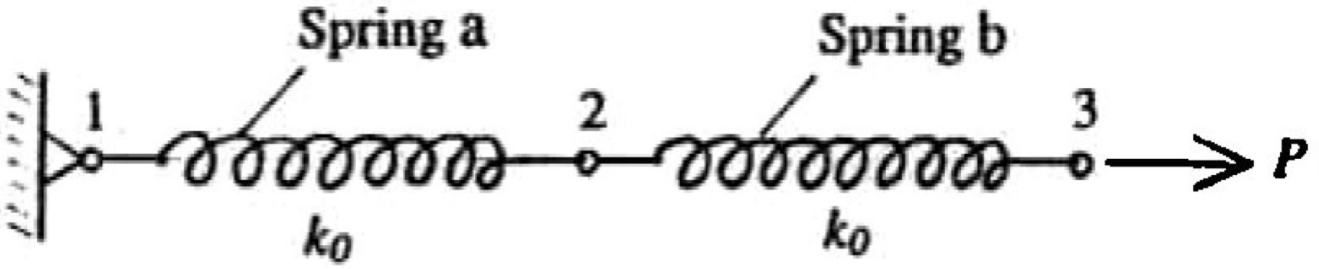


Fig. 1 Simple Statically Determinate Structure

$$D_3 = \int_{l_0} \frac{1\,P}{EA_0}\,dx + \int_{l_0} \frac{1\,P}{EA_0}\,dx$$

$$= \frac{Pl_0}{EA_0} + \frac{Pl_0}{EA_0} = \frac{P}{k_0} + \frac{P}{k_0}$$

静定構造なので、全体変形 $\frac{2Pl_0}{EA_0}$ の内、a が $\frac{Pl_0}{EA_0}$ 、b が $\frac{Pl_0}{EA_0}$ だけ変形寄与したと考えて良い。

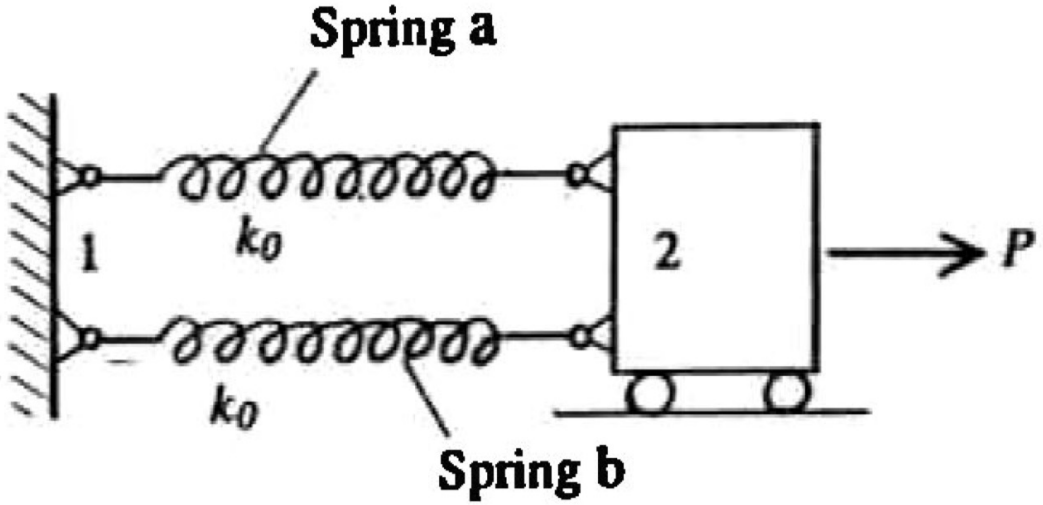


Fig. 3 Simple Statically Indeterminate Structure

$$D_2 = \int_{l_0} \frac{\frac{1}{2}\frac{P}{2}}{EA_0} dx + \int_{l_0} \frac{\frac{1}{2}\frac{P}{2}}{EA_0} dx = \frac{P l_0}{4EA_0} + \frac{P l_0}{4EA_0}$$

大胆な考え：不静定構造の場合でも、全体変形 $\frac{Pl_0}{2EA_0}$ の内、a が $\frac{Pl_0}{4EA_0}$ 、b が $\frac{Pl_0}{4EA_0}$ だけ変形寄与したと考える。

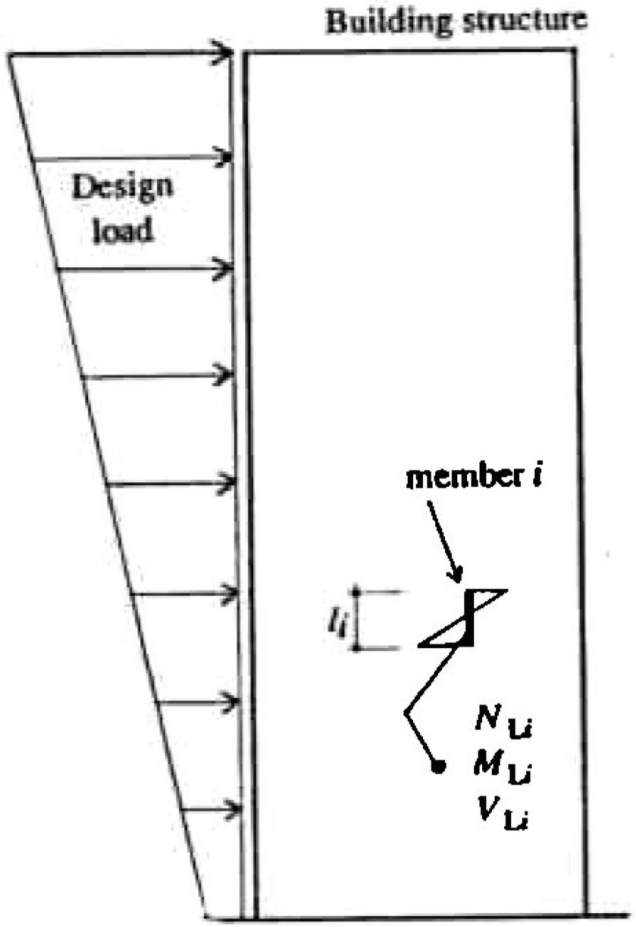


Fig. 4.a Stress Resultants Caused by Design Load

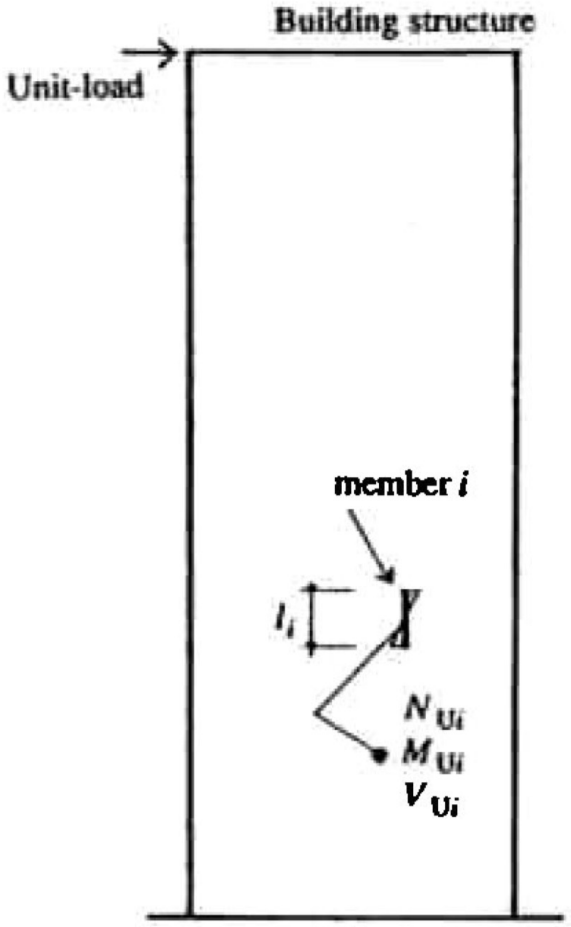


Fig. 4.b Stress Resultants Caused by Unit Load

$$D_{top} = \sum_{i=1}^{m} d_i$$

where

$$d_i = \frac{N_{Ui} N_{Li}}{EA_i} l_i + \int_{l_i} \frac{M_{Ui} M_{Li}}{EI_i} dx + \frac{\alpha V_{Ui} V_{Li}}{GA_i} l_i$$

$$W_{total} = \sum_{i=1}^{m} w_i$$

部材 I の幅をαi 倍に広げ、全重量の変更はしないとすると、

$$D'_{top} = \sum_{i=1}^{m} \frac{d_i}{\alpha_i}$$

$$W_{total} = W'_{total} = \sum_{i=1}^{m} \alpha_i w_i$$

LaGrange Multiplier λ を用いて

$$D'_{top} = \sum_{i=1}^{m} \frac{d_i}{\alpha_i} + \lambda \left(\sum_{i=1}^{m} (\alpha_i w_i) - W_{total} \right)$$

偏微分をゼロとおくと、頂部の変形を最小化するための個々の w'i とαiが求まる

$$w'_i = \alpha_i w_i = \frac{\sqrt{d_i w_i}}{\sum_{j=1}^{m} \sqrt{d_j w_j}} W_{total}$$

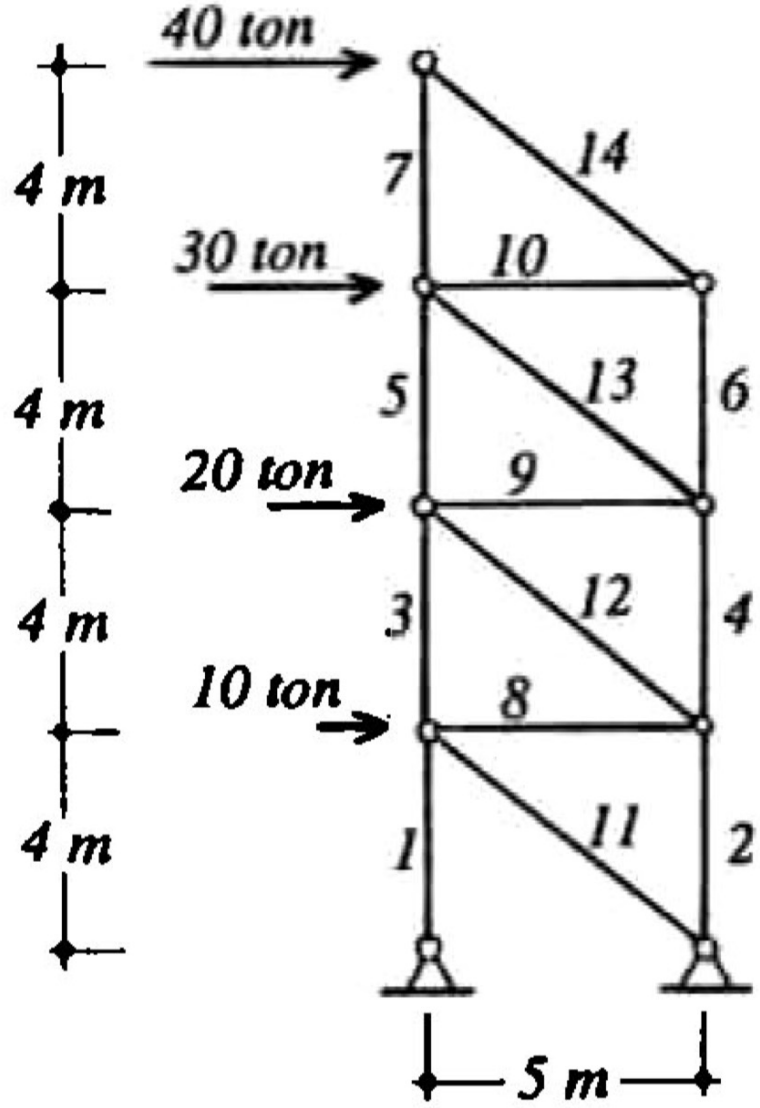


Fig. 8 Truss Structure

n	i	j	L	Wi	AE	AvG/α	EI	Na	Qa	Mai	Maj	Nu	Qu	Mui	Muj
			材長cm	全重量ton				設計荷重時＞				単位荷重時＞			
総和＞				3.79											
1	1	3	400	0.314	210000	1	1	240.00	0.00	0.00	0.00	3.200	0.000	0.000	0.000
2	2	4	400	0.314	210000	1	1	160.00	0.00	0.00	0.00	2.400	0.000	0.000	0.000
3	3	5	400	0.314	210000	1	1	160.00	0.00	0.00	0.00	2.400	0.000	0.000	0.000
4	4	6	400	0.314	210000	1	1	88.00	0.00	0.00	0.00	1.600	0.000	0.000	0.000
5	5	7	400	0.314	210000	1	1	88.00	0.00	0.00	0.00	1.600	0.000	0.000	0.000
6	6	8	400	0.314	210000	1	1	32.00	0.00	0.00	0.00	0.800	0.000	0.000	0.000
7	7	9	400	0.314	210000	1	1	32.00	0.00	0.00	0.00	0.800	0.000	0.000	0.000
8	3	4	500	0.196	105000	1	1	90.00	0.00	0.00	0.00	1.000	0.000	0.000	0.000
9	5	6	500	0.196	105000	1	1	70.00	0.00	0.00	0.00	1.000	0.000	0.000	0.000
10	7	8	500	0.196	105000	1	1	40.00	0.00	0.00	0.00	1.000	0.000	0.000	0.000
11	2	3	640.3	0.251	105000	1	1	128.06	0.00	0.00	0.00	1.281	0.000	0.000	0.000
12	4	5	640.3	0.251	105000	1	1	115.26	0.00	0.00	0.00	1.281	0.000	0.000	0.000
13	6	7	640.3	0.251	105000	1	1	89.64	0.00	0.00	0.00	1.281	0.000	0.000	0.000
14	8	9	640.3	0.251	105000	1	1	51.23	0.00	0.00	0.00	1.281	0.000	0.000	0.000

NaNuL/EA	α QaQuL/GAv	Bendng	Di	DiWi	SQ(DiWi)	S/Sum(S)	Wi'=Wt*S/Sm	Wi'/Wi	D'i
			和と**現**					α	和と**新**
軸変形寄与	剪断変形寄与	曲げ寄与	寄与変位	過程1	過程2	過程3	修正後の重量	修正係数	寄与変位
			7.513		4.91539		3.790		**6.375**
1.463	0.000	0.000	1.463	0.45934	0.67774	0.13788	0.523	**1.664**	**0.879**
0.731	0.000	0.000	0.731	0.22967	0.47924	0.09750	0.370	**1.177**	**0.622**
0.731	0.000	0.000	0.731	0.22967	0.47924	0.09750	0.370	**1.177**	**0.622**
0.268	0.000	0.000	0.268	0.08421	0.29019	0.05904	0.224	**0.713**	**0.376**
0.268	0.000	0.000	0.268	0.08421	0.29019	0.05904	0.224	**0.713**	**0.376**
0.049	0.000	0.000	0.049	0.01531	0.12374	0.02517	0.095	**0.304**	**0.160**
0.049	0.000	0.000	0.049	0.01531	0.12374	0.02517	0.095	**0.304**	**0.160**
0.429	0.000	0.000	0.429	0.08400	0.28983	0.05896	0.223	**1.140**	**0.376**
0.333	0.000	0.000	0.333	0.06533	0.25560	0.05200	0.197	**1.006**	**0.332**
0.190	0.000	0.000	0.190	0.03733	0.19322	0.03931	0.149	**0.760**	**0.251**
1.000	0.000	0.000	1.000	0.25110	0.50110	0.10194	0.386	**1.539**	**0.650**
0.900	0.000	0.000	0.900	0.22599	0.47538	0.09671	0.367	**1.460**	**0.617**
0.700	0.000	0.000	0.700	0.17577	0.41925	0.08529	0.323	**1.288**	**0.544**
0.400	0.000	0.000	0.400	0.10044	0.31692	0.06448	0.244	**0.974**	**0.411**

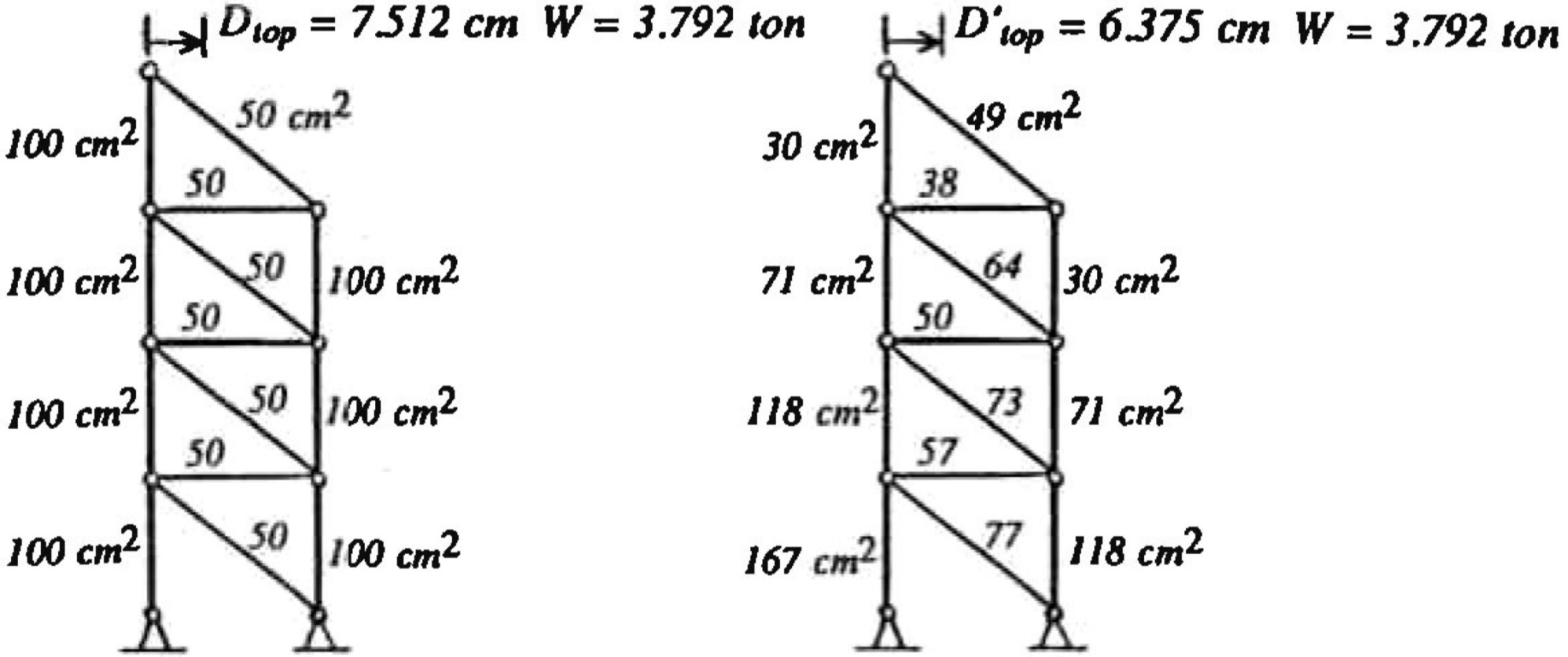
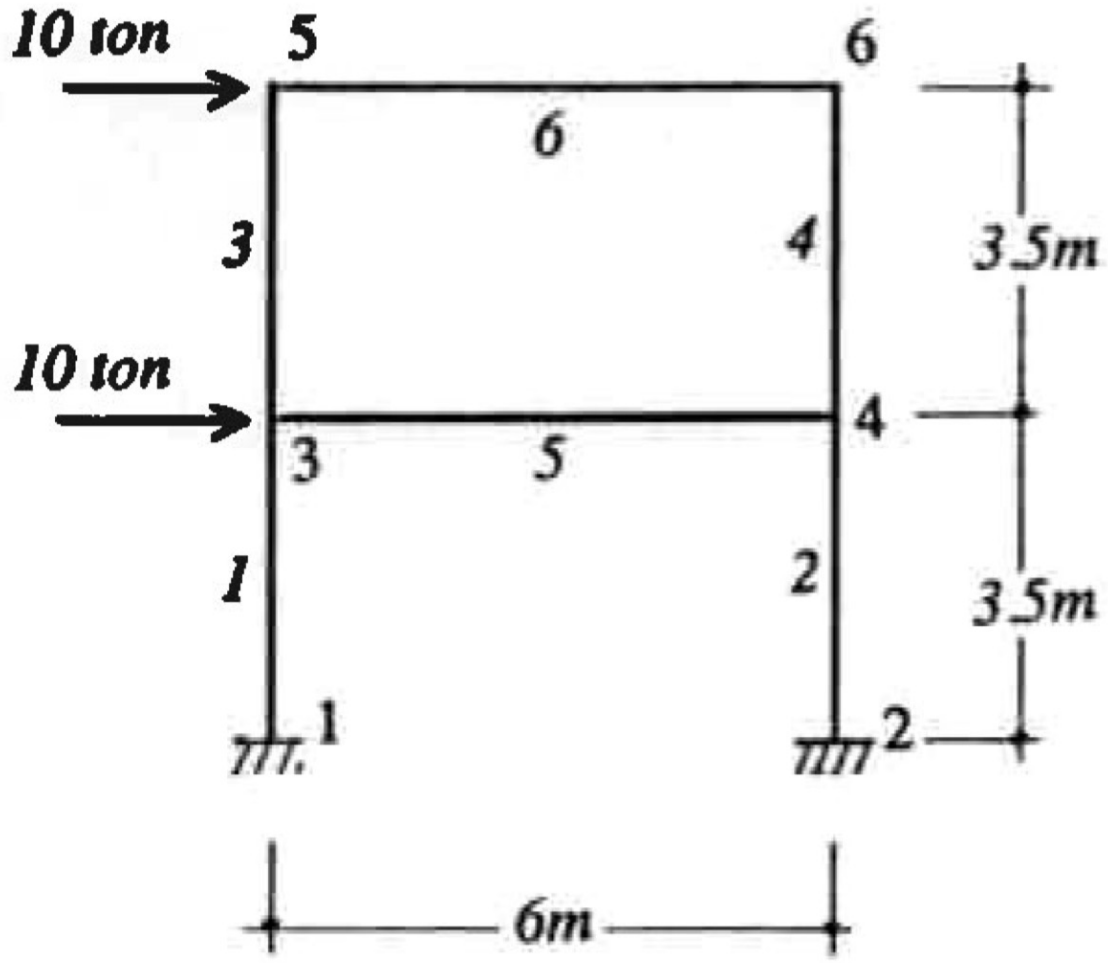


Fig. 9 Sectional Area and D_{top} of Truss Structure



Girder : H-400x200x8x13

Column : H-250x250x9x14

Fig. 6 Simple Frame

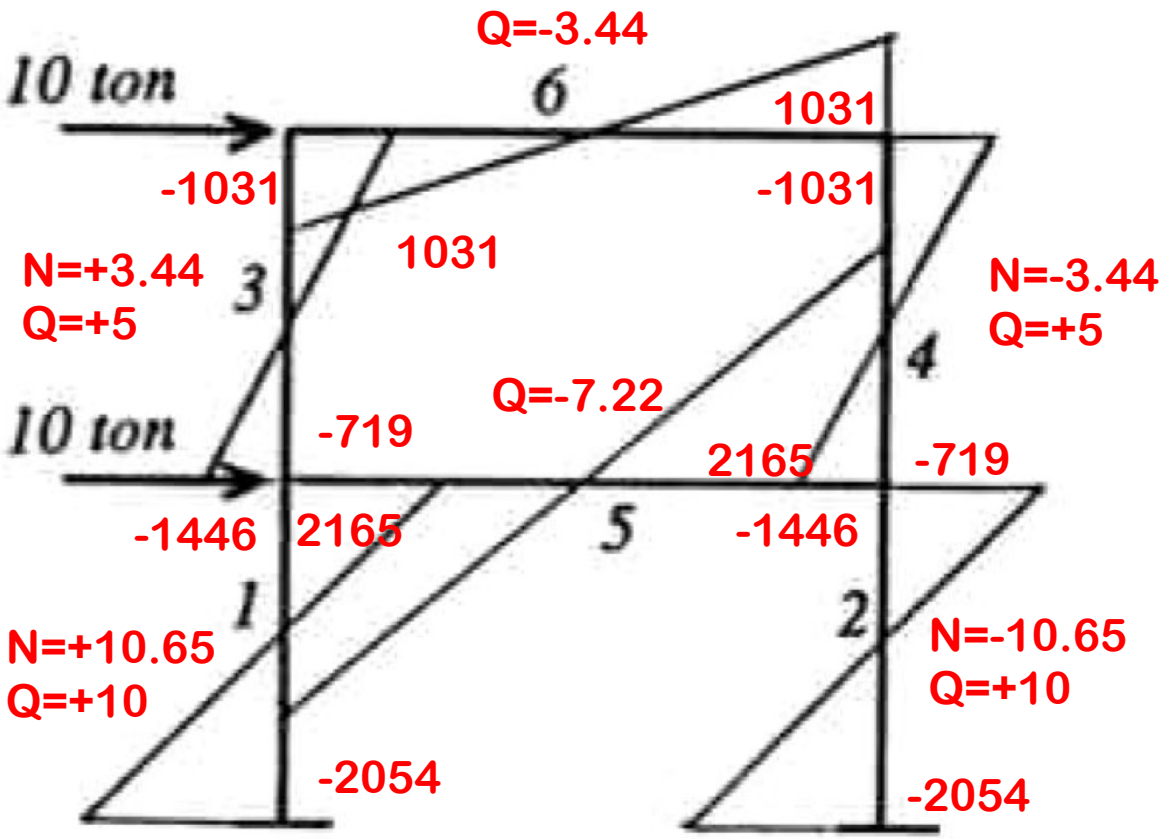


Fig. 7.a Stress Resultants Caused by Design

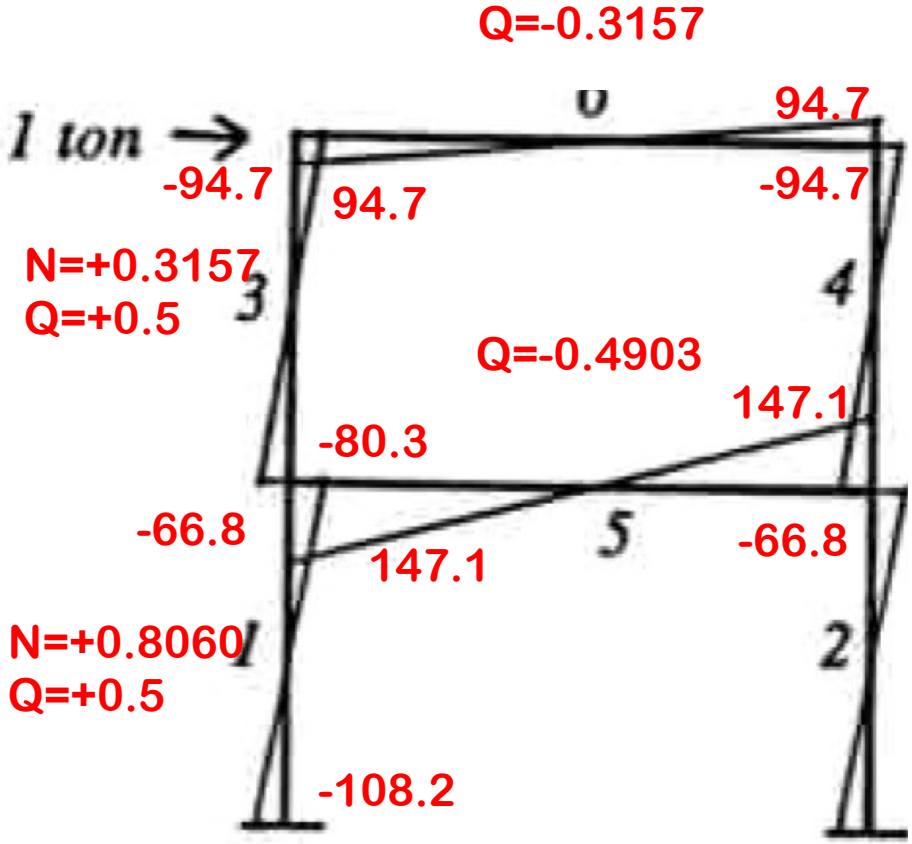


Fig. 7.b Stress Resultants Caused b

n	i	j	L	Wi	AE	AvG/α	EI	Na	Qa	Mai	Maj	Nu	Qu	Mui	Muj
			材長cm	全重量ton				設計荷重時＞				単位荷重時＞			
総和＞				1.7604											
1	1	3	350	0.2472	188958	18225	22215505	10.653	10	-2054.1	-1445.9	0.806	0.5	-108.2	-66.8
2	2	4	350	0.2472	188958	18225	22215505	-10.653	10	-2054.1	-1445.9	-0.806	0.5	-108.2	-66.8
3	3	5	350	0.2472	188958	18225	22215505	3.436	5	-719.2	-1030.8	0.3157	0.5	-80.3	-94.7
4	4	6	350	0.2472	188958	18225	22215505	-3.436	5	-719.2	-1030.8	-0.3157	0.5	-80.3	-94.7
5	3	4	600	0.3858	172032	25920	48226000	0	-7.217	2165	2165	0	-0.4903	147.1	147.1
6	5	6	600	0.3858	172032	25920	48226000	0	-3.436	1030.8	1030.8	0	-0.3157	94.7	94.7

NaNuL/EA	αQaQuL/GAv	Bendng	Di	DiWi	SQ(DiWi)	S/Sum(S)	Wi'=Wt*S/Sm	Wi'/Wi	D'i
			和と**現**					α	和と**新**
軸変形寄与	剪断変形寄与	曲げ寄与	寄与変位	過程1	過程2	過程3	修正後の重量	修正係数	寄与変位
			4.803		2.82638		1.760		**4.538**
0.016	0.096	0.903	1.015	0.25097	0.50097	0.17725	0.312	**1.262**	**0.804**
0.016	0.096	0.903	1.015	0.25097	0.50097	0.17725	0.312	**1.262**	**0.804**
0.002	0.048	0.420	0.470	0.11613	0.34077	0.12057	0.212	**0.859**	**0.547**
0.002	0.048	0.420	0.470	0.11613	0.34077	0.12057	0.212	**0.859**	**0.547**
0.000	0.082	1.321	1.403	0.54114	0.73563	0.26027	0.458	**1.188**	**1.181**
0.000	0.025	0.405	0.430	0.16587	0.40727	0.14410	0.254	**0.658**	**0.654**

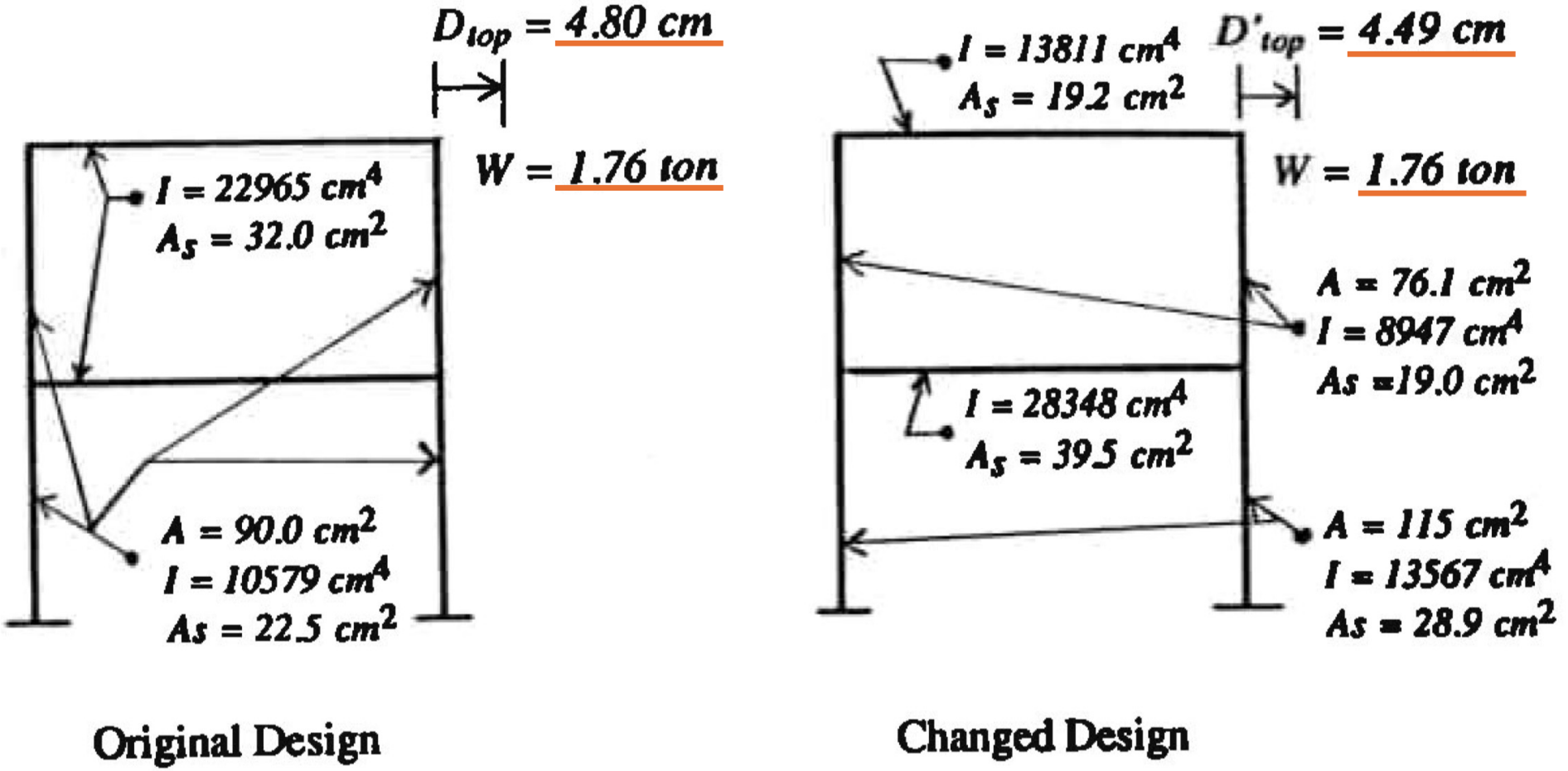


Fig. 10 Sectional Properties and D_{top} of Steel Frame

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Abstract

The sizing of the members of the lateral resistance system for multi-story buildings is often controlled by stiffness requirements. In order to achieve economical buildings, it is important that these members be appropriately sized and that the structural materials be efficiently distributed among the various components. This paper presents a sizing technique utilizing energy methods that is currently in use by several firms involved in the design of high-rise buildings.

Theoretical Basis – Axial Members

Frequently in the design of high-rise buildings, the structural designer wants to use the minimum material to resist a prescribed wind load without exceeding a deflection criteria (such as tip deflection). In essence, the external work done by the wind load has been predefined.

The task then is to proportion the structure so that the internal work is attained with a minimum volume structure. We know from virtual work methods that the deflection at the top of a braced structure *(Fig. 1)* is given by

$$\Delta = \sum\left(\frac{nFL}{EA}\right) \qquad (1)$$

where F is the force in a member due to the lateral (wind) loads, L/EA are the geometric and material properties of a member, and n is the force in the member due to a unit virtual lateral load applied at the top.

Equation (1) contains some very useful information for sizing of members. The FL/EA term gives the actual elongation of the individual member. The n is a

By moving the material from one member to another, it is possible for all members of the structure to have equal energy densities.

Now the question is whether the resulting structure is a minimum volume structure. This can be investigated using LaGrange Multipliers. The approach is to minimize the deflection of the structure as given by

$$f=\sum\frac{nFL}{EA} \qquad (3)$$

subject to the constraint that it has a constant volume of material (V). This constraint can be formulated as

$$g=\Sigma AL-V=0 \qquad (4)$$

A constrained deflection equation may then be written as

$$\Delta=f+\lambda g \qquad (5)$$

Substituting for f an g from Equations (3) and (4) respectively:

$$\Delta=\sum\frac{nFL}{EA}+\lambda(\Sigma AL-V) \qquad (6)$$

where λ is a constant called the LaGrange Multiplier. Given that the geometry of the structure is set, the in-